

Ω mega 2006: The Foundational Change from HOL to DeTSeT

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October 26, 2006

The decision was made at the Ω mega-Tagen in July and August, 2006, to change the foundation of Ω mega from higher-order logic (HOL) as formulated in Church's type theory (extended in Ω mega with partially-ordered sorts) with a dependently typed set theory (DeTSeT). We enumerate some of the reasons for the change.

1. Since HOL can be interpreted in set theory, one can map HOL into DeTSeT. Hence DeTSeT is at least as expressive as HOL.
2. The description of the checker for DeTSeT is simple enough that one can easily write an independent proof checker. At the moment, DeTSeT is implemented in the system Scunak, which can be used as an external, independent proof checker.
3. Proof search in HOL is complicated by the infinite signature of logical constants. In DeTSeT we reduce the signature to a finite number of logical constants.
4. In order to represent many concepts in higher-order logic naturally one must use type variables. The semantics of higher-order logic with type variables is complicated (and largely ignored). Also, in practical terms, there is no reasonable, complete semi-decision procedure for pre-unification of simply typed λ -terms with type variables. Since (object-level) sets can be used as DeTSeT types, there is no need for type variables to express concepts in DeTSeT. Also, since DeTSeT only uses second-order λ -calculus with no type variables, the unification problem becomes simpler and the matching problem is decidable.
5. HOL is based on simple types. DeTSeT is based on dependent types, which allows a greater degree of expressivity.
6. Representing partial functions in HOL is known to be problematic. In DeTSeT we avoid this problem using the fact that every function is total on its domain and every set (a potential domain of a function) can be used as a type.