Verifying and Invalidating Textbook Proofs using Scunak

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Scunak

- Assistant System for Formal Mathematics
- Implemented and Distributed
- Dependently Typed Set Theory

Untyped Set Theory encoded in a

LF-like Dependent Type Theory with

Proof Irrelevance

and some Σ-types so that $Sets \subseteq Classes \subseteq Types$.

A Textbook Proof

First (Sample) Proof in Bartle and Sherbert's *Introduction to Real Analysis*:

Distributivity: $A \cap (B \cup C) = ((A \cap B) \cup (A \cap C))$

In order to give a sample proof, we shall prove the first equation in (d). Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

A Textbook Proof

First (Sample) Proof in Bartle and Sherbert's *Introduction to Real Analysis*:

Distributivity: $A \cap (B \cup C) = ((A \cap B) \cup (A \cap C))$

Conversely, let y be an element of $(A \cap B) \cup (A \cap C)$. Then, either (iii) $y \in A \cap B$, or (iv) $y \in A \cap C$. It follows that $y \in A$, and either $y \in B$ or $y \in C$. Therefore, $y \in A$ and $y \in B \cup C$ so that $y \in A \cap (B \cup C)$. Hence $(A \cap B) \cup (A \cap C)$ is a subset of $A \cap (B \cup C)$.

A Textbook Proof

First (Sample) Proof in Bartle and Sherbert's *Introduction to Real Analysis*:

Distributivity: $A \cap (B \cup C) = ((A \cap B) \cup (A \cap C))$

In view of Definition 1.1.1, we conclude that the sets $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are equal.

A Textbook Proof

First (Sample) Proof in Bartle and Sherbert's *Introduction to Real Analysis*:

Distributivity: $A \cap (B \cup C) = ((A \cap B) \cup (A \cap C))$

In view of Definition 1.1.1, we conclude that the sets $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are equal.

Claim: This is a Human Readable Natural Deduction Proof.

Approach: Verify by Interpreting the Text as Instructions for Constructing a Natural Deduction Proof (Proof Term)

Scunak

Can Verify the Textbook Proof

Can Reject Mutilated Versions of the Textbook Proof

Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

Mutilated Proofs

Let x be an element of $A \cup (B \cap C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

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Let x be an element of $A \cup (B \cap C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

Scunak Diagnosis:

Let x be an element of $A \cup (B \cap C)$

The type you gave for x does not seem to be correct.

Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

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Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in$ $\boxed{A}$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

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Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in$ B and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

Scunak Diagnosis:
then $x \in B$ and $x \in B \cup C$
I am not sure that statement follows, Chad.

Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in$ $\boxed{A}$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

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Scunak Diagnosis:

so $x \in (A \cap B) \cup (A \cap C)$

I am not sure that statement follows, Chad.

Mutilated Proofs

Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$. This means that $x \in A$, and either $x \in B$ or $x \in C$. Hence we either have (i) $x \in A$ and $x \in B$, or we have (ii) $x \in A$ and $x \in C$. Therefore, either $x \in A \cap B$ or $x \in A \cap C$, so $x \in (A \cap B) \cup (A \cap C)$. This shows that $A \cap (B \cup C)$ is a subset of $(A \cap B) \cup (A \cap C)$.

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Scunak Diagnosis:
This shows that $A \cap (B \cup C)$ is a subset of $(A \cup C) \cap (B \cup C)$
We have not shown that.

Extracting Logical Content

In order to give a sample proof, we shall prove the first equation in (d). Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.

Scan Until a Context-Free Grammar Rule Matches

First Sentence Matches No Rule $\Rightarrow$ Ignore

Extracting Logical Content

In order to give a sample proof, we shall prove the first equation in (d). Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.

Rule:

$$LET \texttt{\$}MATHVARIABLE\texttt{\$}\ \texttt{be}\ A\ ATTRIBUTION$$

Extracting Logical Content

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Rule:

$$LET\ \$MATHVARIABLE\$\ \texttt{be}\ A\ ATTRIBUTION$$

Representation as Scunak Command:

```
let x:(in (A \cap (B \cup C))).
```

Extracting Logical Content

In order to give a sample proof, we shall prove the first equation in (d). Let x be an element of $A \cap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.

Representation as Scunak Command:

```
hence ((x::A) & (x::(B \cup C))).
```

Representation as Scunak Commands

```
1. let x:(in (A \cap (B \cup C))).
2. hence ((x::A) & (x::(B \cup C))).
3. hence ((x::A) & ((x::B) | (x::C))).
4. clearly (((x::A) & (x::B)) | ((x::A) &
   (x::C))).
5. hence ((x::(A \cap B)) | (x::(A \cap C))).
6. hence (x::((A \cap B) \cup (A \cap C))).
7. hence ((A \cap (B \cup C)) <= ((A \cap B)
   \cup (A \cap C))).
```

Representation as Scunak Commands

```
 8. let y:(in ((A \cap B) \cup (A \cap C))).
 9. clearly ((y::(A \cap B)) | (y::(A \cap
    C))).
10. hence ((y::A) & ((y::B) | (y::C))).
11. hence ((y::A) & (y::(B \cup C))).
12. hence (y::(A \cap (B \cup C))).
13. hence (((A \cap B) \cup (A \cap C)) <= (A
    \cap (B \cup C))).
14. clearly ((A \cap (B \cup C))==((A \cap B)
    \cup (A \cap C)))
```

Alternative Proof States

Proof State = Open Proof Term + Stack of Tasks

Task = either

Open Task $\langle X, \Gamma, G\rangle$ (X Proof Variable, want instantiation with $\Gamma \vdash X : G$)

or Closed Task $\langle \Gamma, G\rangle$ (bookkeeping)

Initial Proof State: X with Task $\langle X, \Gamma, G\rangle$

$\Gamma = A : \texttt{set}, B : \texttt{set}, C : \texttt{set}$

$G = \texttt{pf}\ \ ((A \cap (B \cup C) {=}{=} (A \cap B) \cup (A \cap C)))$

Two Sets of Rules

Eager Rules Introduce Alternative Proof States to Predict

Usable Rules Are Used to Jusitfy Gaps

Search for Filling Gaps is Very Limited

Rough Idea: Break Down And's and Or's, then apply a Usable Rule

Example of an Eager Rule: Set Extensionality

Using an Eager Rule

Using Eager Rule of Set Extensionality, obtain Open Proof Term using

$$X \mapsto (\texttt{setextsub}...YZ)$$

where Y and Z are Variables

$$Y : \texttt{pf} \;\; (A \cap (B \cup C)) \subseteq ((A \cap B) \cup (A \cap C))$$

$$Z : \texttt{pf} \;\; (((A \cap B) \cup (A \cap C)) \subseteq (A \cap (B \cup C)))$$

Five Initial Proof States

Let Γ be $A : \texttt{obj}, B : \texttt{obj}, C : \texttt{obj}$.

Let G be $\texttt{pf}\ \ ((A \cap (B \cup C)) {=}{=} ((A \cap B) \cup (A \cap C)))$.

Let $G1$ be $\texttt{pf}\ \ ((A \cap (B \cup C)) \subseteq ((A \cap B) \cup (A \cap C)))$.

Let $G2$ be $\texttt{pf}\ \ (((A \cap B) \cup (A \cap C)) \subseteq (A \cap (B \cup C)))$.

			$\langle Y, \Gamma, G1 \rangle$	$\langle Z, \Gamma, G2 \rangle$
	$\langle Y, \Gamma, G1 \rangle$	$\langle Z, \Gamma, G2 \rangle$	$\langle Z, \Gamma, G2 \rangle$	$\langle Y, \Gamma, G1 \rangle$
$\langle X, \Gamma, G \rangle$	$\langle Z, \Gamma, G2 \rangle$	$\langle Y, \Gamma, G1 \rangle$	$\langle \Gamma, G \rangle$	$\langle \Gamma, G \rangle$
①	②	③	④	⑤

Five Initial Proof States

$LHS ::== (A \cap (B \cup C))$

$RHS ::== ((A \cap B) \cup (A \cap C))$

$\vdash^? LHS = RHS$

①

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

②

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

③

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

④

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

⑤

Analyzing the First Step

Textbook Version: Let x be an element of $A \cap (B \cup C)$

Scunak Version: let x:(in (A \cap (B \cup C))).

Idea: Try to perform a step corresponding to this let in each alternative proof state.

Find a Usable Rule which can Conclude the Goal with a Premiss Parametric in $x \in A \cap (B \cup C)$.

In Alternative (1) the top task is to instantiate X, i.e., prove $((A \cap (B \cup C)) = ((A \cap B) \cup (A \cap C)))$

No Usable Rule introduces an $x \in A \cap (B \cup C)$.

Analyzing the First Step

Textbook Version: Let x be an element of $A \cap (B \cup C)$

Scunak Version: let x:(in (A \cap (B \cup C))).

Idea: Try to perform a step corresponding to this let in each alternative proof state.

Find a Usable Rule which can Conclude the Goal with a Premiss Parametric in $x \in A \cap (B \cup C)$.

In Alternatives (2) and (4) the top task is to instantiate Y, i.e., prove $((A \cap (B \cup C)) \subseteq ((A \cap B) \cup (A \cap C)))$

Subset Introduction Rule Proves This with Premiss $x \in ((A \cap B) \cup (A \cap C))$ Parametric in $x \in A \cap (B \cup C)$.

Analyzing the First Step

Textbook Version: Let x be an element of $A \cap (B \cup C)$

Scunak Version: let x:(in (A \cap (B \cup C))).

Idea: Try to perform a step corresponding to this let in each alternative proof state.

Find a Usable Rule which can Conclude the Goal with a Premiss Parametric in $x \in A \cap (B \cup C)$.

For Alternatives (3), and (5), assuming we have some $x \in A \cap (B \cup C)$ in order to prove $(((A \cap B) \cup (A \cap C)) \subseteq (A \cap (B \cup C)))$ is pointless.

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$\vdash^? LHS = RHS$

①

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

②

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

③

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

④

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

⑤

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$\vdash^? LHS = RHS$

(1)

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

(2)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

(3)

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

(2)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

(3)

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$\vdash^? LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
②

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
③

$\vdash^? LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
$\vdash LHS = RHS$
④

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
$\vdash LHS = RHS$
⑤

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^? x \in RHS$
$\vdash^? RHS \subseteq LHS$
(2a)

$x \in LHS \vdash^? x \in RHS$
$\vdash LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
(2b)

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
(3)

$\vdash^? LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
$\vdash LHS = RHS$
(4)

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
$\vdash LHS = RHS$
(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^? x \in RHS$

$x \in LHS \vdash^? x \in RHS$

$\vdash^? RHS \subseteq LHS$

(2a)

$\vdash LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

(2b)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

(3)

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^? x \in RH$

$x \in LHS \vdash^? x \in RHS$
$\vdash^? RHS \subseteq LHS$
(2a)

$\vdash LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
(2b)

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
(3)

$\vdash^? LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$
$\vdash LHS = RHS$
(4)

$\vdash^? RHS \subseteq LHS$
$\vdash^? LHS \subseteq RHS$
$\vdash LHS = RHS$
(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^{?} x \in RHS$

$x \in LHS \vdash^{?} x \in RHS$
$\vdash^{?} RHS \subseteq LHS$
(2a)

$\vdash LHS \subseteq RHS$
$\vdash^{?} RHS \subseteq LHS$
(2b)

$\vdash^{?} LHS \subseteq RHS$
$\vdash^{?} RHS \subseteq LHS$
$\vdash LHS = RHS$
(4)

$\vdash^{?} RHS \subseteq LHS$
$\vdash^{?} LHS \subseteq RHS$
$\vdash LHS = RHS$
(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^? x \in RHS$

$\vdash LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

(2b)

$x \in LHS \vdash^? x \in RHS$

$\vdash^? RHS \subseteq LHS$

(2a)

$\vdash^? LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^? x \in RHS$

$\vdash^? RHS \subseteq LHS$

(2a)

$x \in LHS \vdash^? x \in RHS$

$\vdash LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

(2b)

$x \in LHS \vdash^? x \in RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4a)

$x \in LHS \vdash^? x \in RHS$

$\vdash LHS \subseteq RHS$

$\vdash^? RHS \subseteq LHS$

$\vdash LHS = RHS$

(4b)

$\vdash^? RHS \subseteq LHS$

$\vdash^? LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

$x \in LHS \vdash^{?} x \in RHS$

$x \in LHS \vdash^{?} x \in RHS$

$\vdash^{?} RHS \subseteq LHS$

(2a)

$\vdash LHS \subseteq RHS$

$\vdash^{?} RHS \subseteq LHS$

(2b)

$x \in LHS \vdash^{?} x \in RHS$

$\vdash^{?} RHS \subseteq LHS$

$\vdash LHS = RHS$

(4a)

$x \in LHS \vdash^{?} x \in RHS$

$\vdash LHS \subseteq$

$\vdash^{?} RHS \subseteq$

$\vdash LHS =$

(4b)

$\vdash^{?} RHS \subseteq LHS$

$\vdash^{?} LHS \subseteq RHS$

$\vdash LHS = RHS$

(5)

Updating Proof States

Let x be an element of $A \cap (B \cup C)$

	$x \in LHS \vdash^? x \in RHS$
$x \in LHS \vdash^? x \in RHS$	$\vdash LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$	$\vdash^? RHS \subseteq LHS$
(2a)	(2b)
	$x \in LHS \vdash^? x \in RHS$
$x \in LHS \vdash^? x \in RHS$	$\vdash LHS \subseteq RHS$
$\vdash^? RHS \subseteq LHS$	$\vdash^? RHS \subseteq LHS$
$\vdash LHS = RHS$	$\vdash LHS = RHS$
(4a)	(4b)

Results

- Indicating Only the 8 Needed Rules as “Usable,” Scunak Checks The Proof in 2 Seconds
- Including All 79 Known Rules (Including “Lemmas”) Concerning Union, Intersection and Subset as “Usable,” Scunak Checks Proof in 62 Seconds
- Including 435 Rules, Scunak Checks Proof in 457 Seconds

Conclusions

To Verify Textbook Proofs a Lightweight Approach May Be Sufficient:

- CFG Covering Basic Linguistic and Math Constructions
- Keep Up With Alternative Stacks of Tasks
- Fill Gaps with Extremely Restricted Search

To Verify Textbook Proofs:

- Full Natural Language Processing Is Not Necessary (?)
- Intermediate Representations (Annotated Text) Is Not Necessary (?)
- Automated Theorem Proving Is Not Necessary (?)